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Stress tensors in p-adic string theory and truncated OSFT

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Abstract: We construct the stress tensors for the p-adic string model and for the pure tachyonic sector of open string field theory by naive metric covariantization of the action. Then we give the concrete energy density of a lump solution of the p-adic model. In the cubic open bosonic string field theory, we also give the energy density of a lump solution and pressure evolution of a rolling tachyon solution.

Keywords: D-branes, String Field Theory.

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Contents

1. Introduction

Much work has been devoted to looking for solutions in string field theory (SFT). Generally speaking, physicists are concerned with two kinds of solutions with different properties. One kind of solutions are the time independent ones which represent the tachyon vacuum or lower dimensional D-branes [1]–[8]. Initiated by Sen [9], time dependent rolling tachyon solutions have recently attracted much attention [10]–[21]. Studying rolling tachyon solutions can give us information about how the tachyon approaches the tachyon vacuum. At the same time, the p-adic model [22], which exhibits a lot of properties of string field theory, is also of interest. In this model, the potential has a stable vacuum and a tachyon. Studying the dynamics of the tachyon may suggest to us what happens in the same situation for the SFT. Furthermore, one also has lump solutions in the p-adic theory which are identified as lower dimensional D-branes [23].

In [18], Moeller and Zwiebach discussed how to construct the stress tensor for the rolling tachyon solution in the p-adic model. They obtained an unambiguous expression for the energy through a generalized Noether procedure. This analysis could not be extended to the pressure calculation, however, as there are ambiguities in that case. Instead, they included the metric in the action and used the definition of stress tensor in general relativity to calculate the pressure. Then they constructed the rolling tachyon solutions for both the p-adic model and open string field theory (OSFT) in the form of series expansions. After that, they calculated the pressure evolution in the p-adic string case.

It is of interest to consider the stress tensor in the case when the scalar field in the p-adic model depends on all the coordinates. Especially, for a lump solution, what is the profile of the energy distribution along the spatial coordinate? Is it the same as what we expect intuitively? Furthermore, in OSFT, it is important to know if the profile of the energy density has the same properties as that in p-adic string theory. Moeller and Zwiebach showed in [18] that the pressure of the rolling solution in p-adic model does not vanish at large times. For the rolling solution in OSFT, it is of interest to test if one gets vanishing pressure asymptotically or not.

In this paper, we first give the stress tensor in a general form for the p-adic model. When our results are specialized to the time dependent solution in p-adic model, they reproduce the results in [18]. A nontrivial lump solution in p-adic model was given in [22, 23]. We construct the energy density of this solution and compare it with that of the lump solution of ordinary ϕ^3 field theory. We find that these two energy densities have similar spatial profiles. Section 3 is devoted to the case of the pure tachyon field in OSFT. We again construct the stress tensor in a general form. The energy density of a solitonic solution [6] is then constructed in subsection 3.1. Finally we calculate the pressure evolution of a rolling tachyon solution [18].

2. p-adic string theory case

In this section, we first construct the stress tensor of the p-adic string theory by varying the metric. We will find that the expression is exactly the same as the one obtained in [18] if we constrain scalar field to only depend on time. We will also consider the case where the tachyon scalar only depends one spatial coordinate. In that situation, one nontrivial solitonic solution was already given [22, 23]. We then calculate the energy density of that solution. The results show that the total energy, integrated over all space, perfectly agrees with the D24 brane tension as expected. The spatial profile of this energy density looks very like the one of the solitonic solution of ordinary ϕ^3 field theory.

2.1 Stress tensor for p-adic model

The p-adic string theory is defined by the action:

$$S=\int d^dx\mathcal{L}=\frac{1}{g_p^2}\int d^dx\left[-\frac{1}{2}\phi p^{-\frac{1}{2}\Box}\phi+\frac{1}{p+1}\phi^{p+1}\right],\qquad \frac{1}{g_p^2}=\frac{1}{g^2}\frac{p^2}{p-1},\tag{2.1}$$

where $\phi(x)$ is a scalar field, p is a prime integer and g is the open string coupling constant. Though the theory makes sense even as $p\to 1$, in most cases, we will consider $p\geq 2$ in this paper. In this action, there is an infinite number of both time and spatial derivatives. One defines:

$$p^{-\frac{1}{2}\Box}\equiv\exp\left(-\frac{1}{2}\ln p\Box\right)=\sum_{n=0}^{\infty}\left(-\frac{1}{2}\ln p\right)^n\frac{1}{n!}\Box^n,\tag{2.2}$$

and

$$\Box=-\frac{\partial^2}{\partial t^2}+\nabla^2.\tag{2.3}$$

Now we include the metric in the action [18]:

$$
\begin{aligned}
S = S_1 + S_2 \;=\; & \frac{1}{g_p^2}\int d^dx\sqrt{-g}\left[-\frac{1}{2}\phi^2+\frac{1}{p+1}\phi^{p+1}\right]- \\
& -\frac{1}{2g_p^2}\sum_{l=1}^{\infty}\left(-\frac{1}{2}\ln p\right)^l\frac{1}{l!}\int d^dx\sqrt{-g}\phi\Box^l\phi\,,
\end{aligned} \tag{2.4}
$$

where we have split the action into two parts: S_1 represents the potential and S_2 represents the kinetic term. After introduction of the metric $\Box$ becomes the covariant D'Alembertian.

$$
\begin{aligned}
B_l \equiv \int d^dx\sqrt{-g}\phi\,\Box^l\,\phi \;=\; & \int d^dx\,\phi\,\partial_{\mu_1}\sqrt{-g}g^{\mu_1\nu_1}\partial_{\nu_1}\frac{1}{\sqrt{-g}}\partial_{\mu_2}\sqrt{-g}g^{\mu_2\nu_2}\partial_{\nu_2}\times \\
& \cdots\times\frac{1}{\sqrt{-g}}\partial_{\mu_l}\sqrt{-g}g^{\mu_l\nu_l}\partial_{\nu_l}\,\phi\,.
\end{aligned} \tag{2.5}
$$

The stress tensor is given by:

$$
T_{\alpha\beta} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\alpha\beta}}\,. \tag{2.6}
$$

The variation of the potential S_1 in (2.4) contributes:

$$
\frac{2}{\sqrt{-g}}\frac{\delta S_1}{\delta g^{\alpha\beta}} = -\frac{1}{g_p^2}\left(-\frac{1}{2}\phi^2+\frac{1}{p+1}\phi^{p+1}\right)g_{\alpha\beta}, \tag{2.7}
$$

where we have set the metric to be flat with signature $(-,+,+,\ldots,+)$ after the variation and we will use the same convention in the rest of this paper. As for the variation of the kinetic term S_2 in (2.4), from (2.5), we need to vary both factors of $\sqrt{-g}$ and $g^{\mu_i\nu_i}$ with respect to $g^{\alpha\beta}$. First consider varying factors of $\sqrt{-g}$ in (2.5) with respect to $g^{\alpha\beta}$:

$$
\begin{aligned}
\frac{\delta B_l}{\delta\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\alpha\beta}} \;=\; & g^{\mu_1\nu_1}g^{\mu_2\nu_2}\cdots g^{\mu_l\nu_l}\Big(\phi_{\mu_1}\phi_{\nu_1\mu_2\nu_2\cdots\mu_l\nu_l}+\phi_{\mu_1\nu_1}\phi_{\mu_2\nu_2\cdots\mu_l\nu_l}+\cdots \\
& \cdots+\phi_{\mu_1\nu_1\mu_2\nu_2\cdots\mu_l}\phi_{\nu_l}\Big)g_{\alpha\beta}\,,
\end{aligned} \tag{2.8}
$$

with the definition:

$$
\phi_{\mu_1\nu_1\mu_2\nu_2\cdots\mu_l\nu_l} \equiv \partial_{\mu_1}\partial_{\nu_1}\partial_{\mu_2}\partial_{\nu_2}\cdots\partial_{\mu_l}\partial_{\nu_l}\phi(x)\,.
$$

The variation of the factors of $g^{\mu_i\nu_i}$ in (2.5) with respect to $g^{\alpha\beta}$ contributes:

$$
\begin{aligned}
\frac{\delta B_l}{\delta g^{\mu_i\nu_i}}\frac{\delta g^{\mu_i\nu_i}}{\delta g^{\alpha\beta}} \;=\; & -2g^{\mu_1\nu_1}g^{\mu_2\nu_2}\cdots g^{\mu_{l-1}\nu_{l-1}}\Big(\phi_\alpha\phi_{\beta\mu_1\nu_1\mu_2\nu_2\cdots\mu_{l-1}\nu_{l-1}}+ \\
& +\phi_{\alpha\mu_1\nu_1}\phi_{\beta\mu_2\nu_2\cdots\mu_{l-1}\nu_{l-1}}+\cdots \\
& \cdots+\phi_{\alpha\mu_1\nu_1\mu_2\nu_2\cdots\mu_l}\phi_\beta\Big)\,.
\end{aligned} \tag{2.9}
$$

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So, we can calculate δS_2. Finally, the stress tensor is:

$$\begin{aligned}
T_{\alpha\beta} = & -\frac{1}{g_p^2}\left(-\frac{1}{2}\phi^2 + \frac{1}{p+1}\phi^{p+1}\right) g_{\alpha\beta} - \\
& -\frac{1}{2g_p^2}\sum_{l=1}^{\infty}\left(-\frac{1}{2}\ln p\right)^l \frac{1}{l!}\Big\{ g^{\mu_1\nu_1} g^{\mu_2\nu_2}\cdots g^{\mu_l\nu_l}\Big(\phi_{\mu_1}\phi_{\nu_1\mu_2\nu_2\cdots\mu_l\nu_l} + \\
& \qquad + \phi_{\mu_1\nu_1}\phi_{\mu_2\nu_2\cdots\mu_l\nu_l} + \cdots \\
& \qquad + \phi_{\mu_1\nu_1\mu_2\nu_2\cdots\mu_l}\phi_{\nu_l}\Big) g_{\alpha\beta} - \\
& \qquad - 2g^{\mu_1\nu_1} g^{\mu_2\nu_2}\cdots g^{\mu_{l-1}\nu_{l-1}}\Big(\phi_\alpha\phi_{\beta\mu_1\nu_1\mu_2\nu_2\cdots\mu_{l-1}\nu_{l-1}} + \\
& \qquad + \phi_{\alpha\mu_1\nu_1}\phi_{\beta\mu_2\nu_2\cdots\mu_{l-1}\nu_{l-1}} + \\
& \qquad \cdots + \phi_{\alpha\mu_1\nu_1\mu_2\nu_2\cdots\mu_l}\phi_\beta\Big)\Big\}\,. \qquad (2.10)
\end{aligned}$$

If $\phi(x)$ is only time dependent, in (2.10), each $g^{\mu_i\nu_i}$ contributes one '$-$' sign and the second term in the sum survives only for the component T_{00}. This gives the same results as in [18].

One can also use the following identity[1]

$$\delta e^A = \int_0^1 dt\, e^{tA}(\delta A)e^{(1-t)A}$$

to get an alternative "closed" form of the stress tensor, compared with the series expression (2.10):

$$\begin{aligned}
T_{\alpha\beta} = & \frac{g_{\alpha\beta}}{2g_p^2}\Big\{\phi e^{-k\Box}\phi - \frac{2}{p+1}\phi^{p+1} + k\int_0^1 dt(e^{-kt\Box}\phi)(\Box e^{-k(1-t)\Box}\phi) + \\
& + k\int_0^1 dt(\partial_\mu e^{-kt\Box}\phi)(\partial^\mu e^{-k(1-t)\Box}\phi)\Big\} - \\
& -\frac{k}{g_p^2}\int_0^1 dt(\partial_\alpha e^{-kt\Box}\phi)(\partial_\beta e^{-k(1-t)\Box}\phi), \qquad (2.11)
\end{aligned}$$

where $k \equiv \frac{1}{2}\ln p$.

In the case that $\phi(x)$ only depends on one spatial coordinate, say $x \equiv x^{25}$, the last term in the right hand side of (2.11) vanishes for all the components except for $T_{25,25}$. The energy density is

$$\begin{aligned}
E(x) = T_0^0 = & \frac{1}{2g_p^2}\Big\{\phi e^{-k\partial^2}\phi - \frac{2}{p+1}\phi^{p+1} + k\int_0^1 dt(e^{-kt\partial^2}\phi)(\partial^2 e^{-k(1-t)\partial^2}\phi) + \\
& + k\int_0^1 dt(\partial e^{-kt\partial^2}\phi)(\partial e^{-k(1-t)\partial^2}\phi)\Big\}, \qquad (2.12)
\end{aligned}$$

where $\partial^2 \equiv \partial^2/\partial x^2$.

[1] I thank M. Schnabl for suggesting the use of this identity.

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2.2 Energy of the lump solution

There are some previously known solutions for the p-adic model [22, 23]. One of them is the lump solution:

$$\phi(x) = p^{\frac{1}{2(p-1)}} \exp\left(-\frac{1}{2}\frac{p-1}{p\ln p}x^2\right). \tag{2.13}$$

This solution is interpreted as a D24-brane, where x is the coordinate transverse to the brane. This solution can be generalized to lower dimensional branes [23]. The D-brane tension of this solution is:

$$\begin{aligned} \mathcal{T}_{24} &= -\int dx \mathcal{L}(\phi(x)) = -\int dx \frac{1}{2g_p^2}\frac{1-p}{1+p}\phi^{(p+1)}(x) \\ &= \frac{1}{g_p^2}\frac{p-1}{2(p+1)}p^{\frac{p}{p-1}}\sqrt{\frac{2\pi \ln p}{p^2-1}}\,. \end{aligned} \tag{2.14}$$

Using the identity

$$\exp\left(-a\frac{d^2}{dx^2}\right)\exp(-bx^2) = \frac{1}{\sqrt{1-4ab}}\exp\left(-\frac{bx^2}{1-4ab}\right),$$

from (2.12), we can write down the energy density:

$$E(x) = \frac{p-1}{p+1}\sqrt{\frac{2\pi}{(p^2-1)\ln p}}\,p^{\frac{p}{p-1}}|x|\,\mathrm{Erf}\left[\frac{p-1}{p+1}\sqrt{\frac{p^2-1}{2p\ln p}}\,|x|\right]e^{-\frac{2(p-1)x^2}{(p+1)\ln p}}, \tag{2.15}$$

where $\mathrm{Erf}[x] \equiv \frac{2}{\sqrt{\pi}}\int_0^x dt\, \exp(-t^2)$ is the error function. In figure 1, we plot this energy density (the solid line) for $p = 2$. At $x = 0$ and $x \to \pm\infty$, this energy density vanishes. By solving $\frac{d}{dx}E(x) = 0$ numerically, one can see the energy reaches its maxima at $x \approx \pm 0.9997$. From (2.1), the potential is

$$\frac{1}{2}\phi^2 - \frac{1}{p+1}\phi^{p+1}\,,$$

so, the D-brane vacuum is at $\phi = 1$. Moreover, from (2.13), one gets $\phi = 1$ at $x = \pm\sqrt{2}\ln 2 \approx \pm 0.9803$, which are close to the locations where the energy gets its maxima.

The lump solution (2.13) we are considering here, as we mentioned at the beginning of this subsection, is interpreted as a D-24 brane sharply localized on the hyperplane $x = 0$. Therefore, intuitively one may expect the energy to be sharply localized around $x = 0$. But from figure 1, one can see that the energy is somewhat localised around $x \approx \pm 0.9997$ and reaches a local minimum at $x = 0$.

The total energy is:

$$\int_{-\infty}^{\infty} dx\, E(x) = \frac{1}{g_p^2}\frac{p-1}{2(p+1)}p^{\frac{p}{p-1}}\sqrt{\frac{2\pi\ln p}{p^2-1}} \tag{2.16}$$

which is exactly the same as (2.14). In the limit $p \to 1$, $E(x)$ becomes:

$$\lim_{p\to 1} E(x) = \frac{1}{2g^2}x^2\exp(1-x^2)\,. \tag{2.17}$$

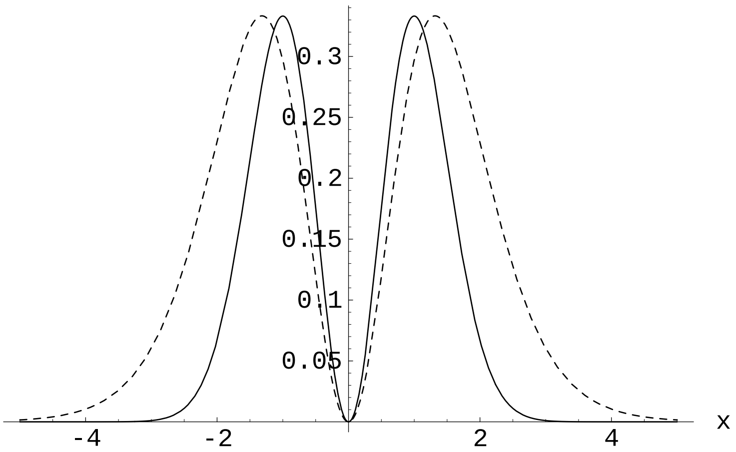

Figure 1: The energy distribution of the lump solution (2.13) of the p-adic model for $p = 2$ (solid line, $g_p^2 E(x)$ versus x) and that (2.18) of ordinary ϕ^3 field theory (dashed line, $g_0^2 E(x)$ versus x).

On the other hand, from (2.1), as $p \to 1$, the action becomes:

$$S = \frac{1}{2g^2} \int d^d x \left(\frac{1}{2} \phi \Box \phi - \frac{1}{2} \phi^2 + \phi^2 \ln \phi \right) .$$

This action has a lump solution:

$$\phi(x) = \exp \left(\frac{1}{2}(1 - x^2) \right),$$

whose energy density is exactly the same as (2.17).

This energy density looks very similar to that of the ordinary ϕ^3 field theory with coupling constant g_0 and unit mass [24]:

$$S = \frac{1}{g_0^2} \int d^d x \left\{ \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} \phi^2 + \frac{1}{3} \phi^3 \right\} ,$$

which has the lump solution:

$$\phi(x) = \frac{3}{2} \left(1 - \tanh^2 \frac{x}{2} \right) , \tag{2.18}$$

with energy density

$$E(x) = \frac{1}{g_0^2} \frac{9}{4} \mathrm{sech}^4 \frac{x}{2} \tanh^2 \frac{x}{2} , \tag{2.19}$$

which is plotted in figure 1 (dashed line).

3. The pure tachyon field of string field theory case

When we expand the string field in the Hilbert space of the first quantized string theory, we can read off the action of the pure tachyonic cubic string field theory. As in the last section, we include the metric in the action and convert all the ordinary derivatives to covariant ones. Variations of the metric again give the stress tensor. Then we calculate the energy density of the lump solution given in [6] and the pressure of the rolling tachyon solution given in [18].

3.1 Stress tensor for the tachyon field in SFT

Firstly, we write down the pure tachyonic action of the cubic SFT. From Sen's conjecture [1], we should add the D-brane tension into the SFT action to cancel the negative energy due to the tachyon. We know that after adding the D-brane tension term to the potential of the cubic SFT, the local minimum of the new potential vanishes [2]. In the same spirit, here we should add a term $\frac{1}{6}K^{-6}$ to the potential to set the local minimum of the potential to zero.

$$S = \frac{1}{g_0^2}\int d^d x\left(\frac{1}{2}\phi^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{3}K^3\widetilde{\phi}^3 - \frac{1}{6}K^{-6}\right), \tag{3.1}$$

where

$$\widetilde{\phi} = \exp\left(\ln K\Box\right)\phi(x) = K^{\Box}\phi(x)\,. \tag{3.2}$$

g_0 is the open bosonic string coupling constant and $K = 3\sqrt{3}/4$. $\Box$ is defined as in the last section. The equation of motion from this action is:

$$K^{-2\Box}(1+\Box)\widetilde{\phi} = K^3\widetilde{\phi}^2\,.$$

In order to separate the term without derivatives from $\widetilde{\phi}(x)$, we define:

$$\begin{aligned}\psi(x) &= \widetilde{\phi}(x) - \phi(x) = \sum_{l=1}^{\infty}\frac{(\ln K)^l}{l!}\Box^l\phi(x)\\ &= \sum_{l=1}^{\infty}\frac{(\ln K)^l}{l!}\frac{1}{\sqrt{-g}}\partial_{\mu_1}\sqrt{-g}g^{\mu_1\nu_1}\partial_{\nu_1}\frac{1}{\sqrt{-g}}\partial_{\mu_2}\sqrt{-g}g^{\mu_2\nu_2}\partial_{\nu_2}\times\cdots\\ &\quad\cdots\times\frac{1}{\sqrt{-g}}\partial_{\mu_l}\sqrt{-g}g^{\mu_l\nu_l}\partial_{\nu_l}\phi(x)\,,\end{aligned} \tag{3.3}$$

where in the last step, we have written the expression in the covariant form. For an arbitrary differentiable function $f(x)$,

$$\int d^d x f(x)\frac{\delta\psi(x)}{\delta g^{\alpha\beta}} = \frac{1}{2}f\psi g_{\alpha\beta} + A_{\alpha\beta}(f) \tag{3.4}$$

where

$$\begin{aligned}A_{\alpha\beta}(f) &= \frac{1}{2}g_{\alpha\beta}\sum_{l=1}^{\infty}\frac{(\ln K)^l}{l!}g^{\mu_1\nu_1}\cdots g^{\mu_l\nu_l}\times\\ &\quad\times\Big(f_{\mu_1}\phi_{\nu_1\mu_2\nu_2\cdots\mu_l\nu_l} + f_{\mu_1\nu_1}\phi_{\mu_2\nu_2\cdots\mu_l\nu_l} + \cdots + f_{\mu_1\nu_1\cdots\mu_l}\phi_{\nu_l}\Big) -\\ &\quad-\sum_{l=1}^{\infty}\frac{(\ln K)^l}{l!}g^{\mu_1\nu_1}\cdots g^{\mu_{l-1}\nu_{l-1}}\Big(f_\alpha\phi_{\beta\mu_1\nu_1\cdots\mu_{l-1}\nu_{l-1}} +\\ &\qquad\qquad + f_{\alpha\mu_1\nu_1}\phi_{\beta\mu_2\nu_2\cdots\mu_{l-1}\nu_{l-1}} +\\ &\qquad\qquad \cdots + f_{\alpha\mu_1\nu_1\cdots\mu_{l-1}\nu_{l-1}}\phi_\beta\Big)\,.\end{aligned} \tag{3.5}$$

Again, we set the metric to be flat with signature $(-1, 1, 1 \cdots 1)$ after the variation. Replace $\widetilde{\phi}$ by $\phi + \psi$ in (3.1), expanding and coupling to the metric:

$$
\begin{aligned}
S = \frac{1}{g_0^2} \int d^dx \sqrt{-g} \Big\{ \Big(\frac{1}{2}\phi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{3} K^3 \phi^3 - \frac{1}{6} K^{-6} \Big) - \\
- K^3 \left(\phi^2 \psi + \phi \psi^2 + \frac{1}{3} \psi^3 \right) \Big\} . \qquad (3.6)
\end{aligned}
$$

Varying the first term in the last right hand side of (3.6) with respect to $\delta g^{\alpha\beta}$ gives

$$
\begin{aligned}
&\frac{1}{g_0^2} \delta_{g^{\alpha\beta}} \int d^dx \sqrt{-g} \Big(\frac{1}{2}\phi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{3} K^3 \phi^3 - \frac{1}{6} K^{-6} \Big) = \\
&\quad = -\frac{g_{\alpha\beta}}{2g_0^2} \left(\frac{1}{2}\phi^2 - \frac{1}{2} (\partial\phi)^2 - \frac{1}{3} K^3 \phi^3 - \frac{1}{6} K^{-6} \right) - \frac{1}{2g_0^2} \partial_\alpha \phi \partial_\beta \phi \\
&\quad \equiv -\frac{1}{g_0^2} C_{\alpha\beta} , \qquad (3.7)
\end{aligned}
$$

where we have defined $C_{\alpha\beta}$ to simplify our notation. As for the second term in the last right hand side of (3.6), note

$$
\begin{aligned}
-K^3 \delta_{g^{\alpha\beta}} \int d^dx \sqrt{-g} \left(\phi^2 \psi + \phi \psi^2 + \frac{1}{3}\psi^3 \right) = \frac{1}{2} K^3 \Big(\phi^2 \psi + \phi \psi^2 + \frac{1}{3}\psi^3 \Big) g_{\alpha\beta} - \\
- K^3 \int d^dx \sqrt{-g}\, \widetilde{\phi}^2 \delta_{g^{\alpha\beta}} \psi .
\end{aligned}
$$

So, from (3.4) and (3.5) the variation of the second term in the last step of (3.6) contributes:

$$
\left(\frac{-K^3}{g_0^2} \right) \delta_{g^{\alpha\beta}} \int d^dx \sqrt{-g} \left(\phi^2 \psi + \phi \psi^2 + \frac{1}{3}\psi^3 \right) = \frac{-K^3}{g_0^2} \left\{ A_{\alpha\beta}(\widetilde{\phi}^2) + \frac{1}{2} \left(\phi \psi^2 + \frac{2}{3} \psi^3 \right) g_{\alpha\beta} \right\} , \qquad (3.8)
$$

Finally, from (3.5), (3.7) and (3.8), the stress tensor is:

$$
T_{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\alpha\beta}} = -\frac{2K^{-3}}{g_0^2} A_{\alpha\beta}(\widetilde{\phi}^2) - \frac{2}{g_0^2} C_{\alpha\beta} . \qquad (3.9)
$$

In the case that $\phi(x)$ only depends on one spatial coordinate, say x^{25}, from (3.5),

$$
A_{\alpha\beta}(\widetilde{\phi}^2) = \sum_{l=1}^{\infty} \frac{(\ln K)^l}{l!} \left\{ \frac{1}{2} g_{\alpha\beta} \sum_{m=1}^{2l-1} \widetilde{\phi}_m^2 \phi_{2l-m} - \delta_{\alpha,25} \delta_{\beta,25} \sum_{m=1}^{l} \widetilde{\phi}_{2m-1}^2 \phi_{2l-2m+1} \right\} . \qquad (3.10)
$$

Plug it into (3.9), we obtain the stress tensor for lump solutions. Similarly, if $\phi(x)$ only depends on time, we can write:

$$
A_{\alpha\beta}(\widetilde{\phi}^2) = -K^3 \sum_{l=1}^{\infty} \frac{(-\ln K)^l}{l!} \left\{ \frac{1}{2} g_{\alpha\beta} \sum_{m=1}^{2l-1} \widetilde{\phi}_m^2 \phi_{2l-m} + \delta_{\alpha,0} \delta_{\beta,0} \sum_{m=1}^{l} \widetilde{\phi}_{2m-1}^2 \phi_{2l-2m+1} \right\} . \qquad (3.11)
$$

Plug it into (3.9), we obtain the stress tensor for rolling solutions

3.2 Energy distribution of the SFT lump solution

In [6], a lump solution of OSFT has been given in the form of an expansion in terms of cosines. We are only concerned with the pure tachyonic mode here, so drop the higher modes:

$$\phi(x) = t_0 + t_1 \cos\left(\frac{x}{R}\right) + t_2 \cos\left(\frac{2x}{R}\right) + \cdots \,, \tag{3.12}$$

where R is the radius of the circle on which the coordinate x is compactified. We can calculate the energy distribution of this solution, from (3.2), (3.3), (3.9) and (3.10):

$$\begin{aligned}
\widetilde{\phi}(x) &= K^{\partial_x^2}\phi(x) = t_0 + t_1 K^{-\frac{1}{R^2}} \cos\left(\frac{x}{R}\right) + t_2 K^{-\frac{4}{R^2}} \cos\left(\frac{2x}{R}\right) + \cdots \,, \\
\psi(x) &= \widetilde{\phi}(x) - \phi(x) = t_1 \left(K^{-\frac{1}{R^2}} - 1\right) \cos\left(\frac{x}{R}\right) + t_2 \left(K^{-\frac{4}{R^2}} - 1\right) \cos\left(\frac{2x}{R}\right) + \cdots \,, \\
E(x) &= T^0_0 = -T_{00} \\
&= -\frac{1}{g_0^2}\left(\frac{1}{2}\phi^2 - \frac{1}{3}K^3\phi^3 - \frac{1}{2}(\partial_x\phi)^2 - \frac{1}{6}K^{-6} + K^3\phi\psi^2 + \frac{2}{3}K^3\psi^3\right) \\
&\quad -\frac{K^3}{g_0^2}\sum_{l=1}^{\infty}\frac{(\ln K)^l}{l!}\sum_{m=1}^{2l-1}\left(\widetilde{\phi}^2\right)_m \phi_{2l-m} \,.
\end{aligned} \tag{3.13}$$

In $R=\sqrt{3}$ case, using the method introduced in [6], one can obtain:

$$t_0 = 0.216046 \,, \qquad t_1 = -0.343268 \,, \qquad t_2 = -0.0978441 \,,$$

when we plug these values into (3.13), we find:

$$\begin{aligned}
E(x) = \frac{1}{g_0^2}\Big(&0.0206937 + 0.0242345 \cos\frac{x}{R} - \\
&- 0.00780954 \cos\frac{2x}{R} - 0.0204855 \cos\frac{3x}{R} - \\
&- 0.0111187 \cos\frac{4x}{R} - 0.00218278 \cos\frac{5x}{R} - 0.000177055 \cos\frac{6x}{R}\Big) \,.
\end{aligned}$$

This lump solution has the interpretation of D24 brane, the tension is:

$$\mathcal{T}_{24} = \int_{-\pi R}^{\pi R} dx \, E(x) \simeq 0.225206 \frac{1}{g_0^2} \,.$$

On the other hand, ϕ=0 is supposed to represent the D25 brane. We have $\mathcal{T}_{25}$=$-V(\phi$=$0) = \frac{1}{6}\frac{K^{-6}}{g_0^2} \simeq 0.0346831\frac{1}{g_0^2}$. Therefore,

$$\frac{1}{2\pi}\frac{\mathcal{T}_{24}}{\mathcal{T}_{25}} \simeq 1.03343$$

a ratio that is unity in string theory.

Figure 2 shows the energy density $E(x)$. As the lump solutions in the p-adic string theory, the energy density is not localised around the hyperplane $x=0$. Instead, $E(x=0)$ is a local minimum. A difference from the p-adic model is that $E(0)$ does not vanish here.

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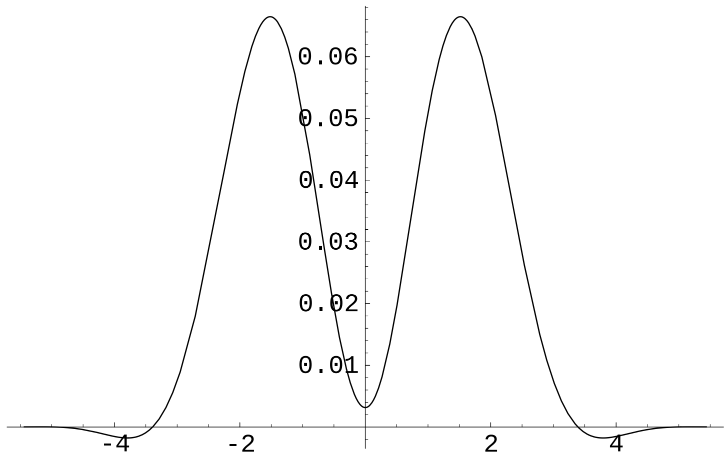

Figure 2: The Energy density of the pure tachyonic lump solution $\phi(x) = t_0 + t_1\cos\frac{x}{R} + t_2\cos\frac{2x}{R}$ of OSFT theory with $R = \sqrt{3}$. the plot is $g_0^2 E(x)$ versus x.

3.3 Pressure evolution of the SFT rolling tachyon solution

In [18], a rolling tachyon solution of OSFT is expressed as a series expansion in $\cosh(nt)$:

$$\phi(t) = t_0 + t_1\cosh t + t_2\cosh 2t + \cdots .$$

From (3.2), (3.3), (3.9) and (3.11):

$$\begin{aligned}
\widetilde{\phi}(t) &= K^{-\partial_t^2}\phi(t) = t_0 + t_1 K^{-1}\cosh t + t_2 K^{-4}\cosh 2t + \cdots , \\
\psi(t) &= \widetilde{\phi}(t) - \phi(t) = t_1\left(K^{-1} - 1\right)\cosh t + t_2\left(K^{-4} - 1\right)\cosh 2t + \cdots , \\
p(t) &= -T_{11} \\
&= \frac{1}{g_0^2}\left(\frac{1}{2}\phi^2 - \frac{1}{3}K^3\phi^3 + \frac{1}{2}\left(\partial_t\phi\right)^2 - \frac{1}{6}K^{-6} + K^3\phi\psi^2 + \frac{2}{3}K^3\psi^3\right) + \\
&\quad + \frac{K^3}{g_0^2}\sum_{l=1}^{\infty}\frac{(-\ln K)^l}{l!}\sum_{m=1}^{2l-1}\left(\widetilde{\phi}^2\right)_m \phi_{2l-m} .
\end{aligned} \tag{3.14}$$

From [18, section 7],

$$t_0 = 0.00162997 , \qquad t_1 = 0.05 , \qquad t_2 = -0.000189714 ,$$

and therefore,

$$\begin{aligned}
p(t) = \frac{1}{g_0^2}\Big(&-0.0346844 + 0.0000416895\cosh t + 0.00124462\cosh 2t - \\
&- 0.0000416042\cosh 3t + 2.59666\times 10^{-7}\cosh 4t - \\
&- 3.97466\times 10^{-10}\cosh 5t + 2.09045\times 10^{-13}\cosh 6t\Big) .
\end{aligned}$$

Figure 3 shows the pressure evolution. It has the same property as the pressure in p-adic theory (figure 10 in [18]).The pressure starts from negative value at time $t = 0$ to force the tachyon roll to the vacuum. But instead of decreasing to zero as $t \to \infty$, it oscillates without bound at large times. So, this solution does not seem to represent tachyon matter.

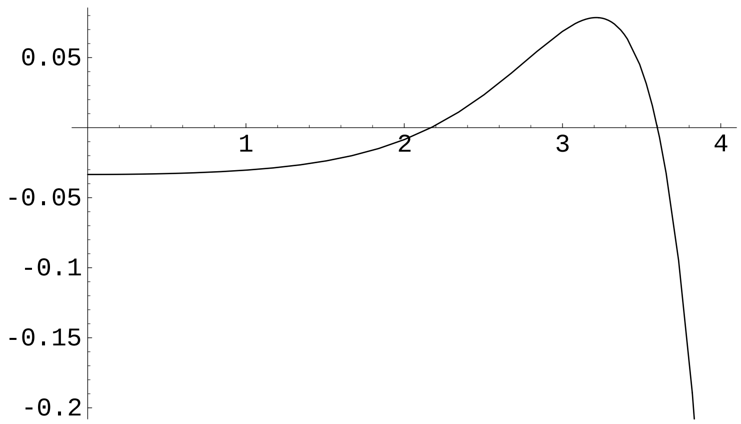

Figure 3: The pressure evolvement of the rolling tachyon solution $\phi(t) = t_0 + t_1 \cosh t + t_2 \cosh 2t$ of OSFT theory. $g_0^2 p(t)$ versus t . As t becomes larger, $p(t)$ oscillates rapidly.

4. Conclusion

By introducing the metric, we have obtained general expressions for the stress tensors both for the p-adic model and for the pure tachyonic sector of open bosonic string field theory [1, 2, 4, 6].

Furthermore, we considered some available solutions and wrote down the corresponding energy densities for space dependent ones and pressure evolutions for time dependent ones. In conformal field theory, D-branes are boundary conditions and one could expect the energy to be sharply localized at the D-brane position. It was not clear whether or not the lumps of the padic string theory would have this property. Our results show that they do not. The energy density vanishes at $x = 0, \pm\infty$. It has two maxima. These two maxima are symmetrically localized with respect to $x = 0$. In the pure tachyonic sector of OSFT, the energy density for the lump solution reaches a local minimum at $x = 0$. For the rolling tachyon solution, the pressure oscillates with growing amplitude instead of asymptotically vanishing. Therefore, as in the p-adic model, the rolling solution we considered in this paper does not seem to represent tachyon matter.

There are two shortcomings of the calculations in OSFT. The first is not including the massive fields. The second is that the coupling of open strings to the metric could have additional terms that vanish in the flat space limit but contribute to the stress tensor. Such phenomena happens in noncommutative field theory [25]. Open-closed string field theory [26] might be needed to calculate the stress tensor with complete confidence. I thank M. Schnabl for bringing this point to my attention.

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